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ABSTRACT

In this work, we examine the three phases of the human hepatic circulatory subsystem in the context of malaria. Here, we suppose that blood is composed of three phases: red blood cells, white blood cells, and plasma. We have used a non-Newtonian power law model to analyze pathological hemoglobin and blood pressure data. We have employed the Navier-Stokes equation of motion and continuity equation in the cylindrical coordinate system. All necessary mathematical expressions are in tensorial form. We have computed the hepatic artery power index numerically. Ultimately, we have established a date-wise linear link between the reduction in blood pressure and the hematocrit.

Keywords: *Hepatic circulation, Three Phases, Non-Newtonian power law, Hemoglobin and Blood Pressure*

INTRODUCTION

The characteristics of blood, one of the body's most complicated and essential fluids, differ significantly from those of conventional, one-phase fluids. Blood is a two- or multi-phase fluid consisting of a liquid (plasma) and a distributed cellular component (mostly red blood cells, white blood cells, and platelets). Blood flow dynamics are especially difficult to forecast due to the many non-Newtonian behaviours and spatial heterogeneities caused by their multi-phase nature. Accurate mathematical and numerical models for blood flow through the vascular system are essential for microcirculation, pathological situations, and medical device design, and they require an understanding of blood as a multi-phase fluid. Plasma, a straw-colored fluid, makes up around 55% of the volume of blood.

The non-Newtonian rheological properties of blood, such as viscoelasticity and shear-thinning viscosity—the viscosity that decreases as the shear rate increases—come from the presence of both plasma and cells. The complexity of these behaviours necessitates the use of two-phase fluid dynamics rather than more traditional Newtonian fluid models. One fluid or a combination of fluids can be mathematically described by two-phase flow models. The homogeneous models incorporate the interaction between the two phases into averaged characteristics such as density and effective viscosity, and they assume that the two phases move with the same velocity field.

A three-phase blood flow model, however, is required to thoroughly examine blood behaviour, particularly in microcirculation, where different components, such as plasma, red blood cells (RBCs), and white blood cells (WBCs), are present.

Function of Hepatic Artery

Sporozoites must get to hepatocytes, where liver stage development takes place, after an infectious mosquito bite. Even though sporozoites are few in number and have a complicated and dangerous dermis-to-hepatocyte journey [1], a single infectious mosquito bite can cause a liver infection, which creates the conditions for successful host colonisation. Unlike the parasite's ookinete stage, which only passes through the mosquito's midgut's basal lamina², and merozoites, which only infiltrate red blood cells, the sporozoite is a distinct invasive stage. Three sporozoites must move through and enter cells since they are extremely motile [2]. They enter the bloodstream through a dermal capillary, travel to the sinusoids, which are the hepatic capillary network, and then pass through the endothelium to reach the hepatic parenchyma. In this case, sporozoites move through several hepatocytes before undergoing a functional change that enables them to enter and settle inside a hepatocyte. The micronemes and rhoptries, a special class of invasive organelles whose constituent

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proteins mediate molecular interactions with the host cell, are released by the sporozoite both during and after invasion. Much less is known about sporozoite infection of hepatocytes than about the molecular mechanisms and sequence of events in cell invasion in relation to merozoite invasion of red blood cells [3].

Real Model:

Frame of reference: - In this model we have apply Navier-Stoke's equation and the equation of continuity, and chosen an orthogonal curvilinear generalised three-dimensional coordinate system, denoted by E3. All blood flow-related quantities are written in tensorial form, which is relatively more realistic. Let P be any point in space with co-ordinate x^i with respect to axes OX^i, O as origin, where i = 1, 2, 3. In time t suppose $v^k = v^k(x^i, t)$ be the blood's velocity, $p = p(x^i, t)$ the thermodynamical pressure, and $\rho = \rho(x^i, t)$ density. Because blood vessels are cylindrical, the governing equations must be converted into a cylindrical coordinate system.

Let *X*= fraction of volume of red blood cells

Y= fraction of volume of white blood cells

Z=1-X-Y= fraction of volume of WBC

Assume that m_1 and m_2 are the mass ratios of RBC and WBC to plasma, respectively then

$m_1 = \frac{X\rho_c}{Z\rho_p}$, $m_2 = \frac{Y\rho_w}{Z\rho_p}$,

Where ρ_c, ρ_p and ρ_w are the densities of RBC, plasma and WBC respectively. Then ρ_m given as

$\frac{1+m1+m2}{\rho_m} = \frac{m_1}{\rho_c} + \frac{1}{\rho_p} + \frac{m_2}{\rho_w}$ where $\rho_m = X\rho_c + Y\rho_w + (1\text{-}X\text{-}Y)\rho_p$

Boundary Conditions

i. The blood flow velocity reaches its greatest value, represented by the equation $v = v_0$, at the point where the radial distance from the blood vessel axis is zero (r = 0).
ii. At r = R, where R is the blood vessel's radius, the blood flow velocity on the blood vessel wall will be zero.

The Continuity Equation

Continuity equation

$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c v^i)_{,i} = 0$ (1)

$\frac{\partial(Y\rho_w)}{\partial t} + (Y\rho_w v^i)_{,i} = 0$ (2)

$\frac{\partial(1-X-Y)\rho_p}{\partial t} + \left((1-X-Y)\rho_p v^i\right)_{,i} = 0$ (3)

Where, v is the velocity of mixture of blood.

Combined equation (1) and (2) and using (3) we get

$\frac{\partial\rho_m}{\partial t} + (\rho_m v^i)_{,i} = 0$ (4)

Equation of motion for blood flow with the three phases

Assuming that the consistency coefficient (also known as the viscosity coefficient) of red blood cells is η_c and applying the force conservation (or momentum conservation) principle in hepatic arteries

$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^j)v^i_{,j} = -Xp_{,j}g^{ij} + X\eta_c\left(g^{jk}v^i_{,k}\right)_{,j}$ (5)

$Y\rho_w \frac{\partial v^i}{\partial t} + (Y\rho_w v^j)v^i_{,j} = -Y\,p_{,j}g^{ij} + Y\eta_w\left(g^{jk}v^i_{,k}\right)_{,j}$ (6)

And $(1\text{-}X - Y)\rho_p \frac{\partial v^i}{\partial t} + \left\{(1-X-Y)\rho_p v^j\right\}v^i_{,j} = -(1-X-Y)\,p_{,j}g^{ij} + (1-X-Y)\eta_p\left(g^{jk}v^i_{,k}\right)_{,j}$ (7)

Now adding (5) and (6) and using (7) then equation of motion for blood flow will be

$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j)v^i_{,j} = -p_{,j}g^{ij} + \eta_m\left(g^{jk}v^i_{,k}\right)_{,j}$ (8)

Where $\eta_m = X\eta_c + Y\eta_w + (1-X-Y)\eta_p$ is the viscosity coefficient of blood as a mixture of red blood cells, white blood cells, and plasma.

Let X be the volume fraction of the unit volume covered by red blood cells, H/100, where H is the haematocrit; let Y be the volume portion covered by white blood cells; and let (1-X-Y) be the volume portion covered by plasma.

It is necessary to convert the above governing equations into cylindrical coordinates since blood vessels are cylindrical. If $(\ ,\theta,z)$ is cylindrical coordinate then matrix of metric tensor and conjugate metric tensor respectively

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Christoffel's symbols of 2nd kind

$$\left\{\begin{matrix} 1 \\ 2\ \ 2 \end{matrix}\right\} = -r\ ,\ \left\{\begin{matrix} 2 \\ 2\ \ 1 \end{matrix}\right\} = \left\{\begin{matrix} 2 \\ 1\ \ 2 \end{matrix}\right\} = \frac{1}{r}$$

Except of these all are zero.

Physical components relation

$$\sqrt{g_{11}}v^1 = v_r \quad or, v_r = v^1$$
$$\sqrt{g_{22}}v^2 = v_\theta \quad or, v_\theta = rv^2$$
$$\sqrt{g_{22}}v^3 = v_z \quad or, v_z = v^3$$

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and the physical component of $-p_{,q}\, g^{pq} are -\sqrt{g_{pp}} p_{,q}\, g^{pq}$

The matrix of physical component of shearing stress – tensor

$\tau^{,pq} = \eta_m (e^{pq})^n = \eta_m (g^{pr} v^p_{,r} + g^{qr} v^q_{,r})^n$ will be:

$$\tau'^{pq} = \begin{bmatrix} 0 & 0 & \eta_m \left(\frac{\partial v_r}{\partial z}\right)^n \\ 0 & 0 & 0 \\ \eta_m \left(\frac{\partial v_z}{\partial r}\right)^n & 0 & 0 \end{bmatrix}$$

The covariant derivative of $T^{,pqj}$ is

$$\tau^{,pq}_{,q} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^q} \left(\sqrt{g} \tau^{,pq}\right) + \left\{ q \; p \; r \right\} \tau^{,rq} \quad (9)$$

In the context of the above facts, the equation of motion and the equation of continuity are converted into the following cylindrical coordinate system:

The equation of continuity –

$$\frac{\partial v}{\partial z} = 0 \quad (10)$$

The equation of motion –

$$-\frac{\partial p}{\partial r} = 0, \quad (11)$$

$$0 = 0, \quad (11a)$$

$$0 = -\frac{\partial p}{\partial r} + \frac{\eta_m}{r} \frac{\partial}{\partial r} \left(r \left(\frac{dv}{dr} \right)^n \right) \quad (12)$$

Here we have taken in view that flow is axial symmetric in artery then

$v_\theta = 0 \; and v_r, \; v_z \; and \; p$ independent of θ. Also

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$$

From (10), we get $v_z = v(r)$ (13)

Equation (11) implies that $p = p(z)$ (14)

Using (13) and (14) in (12) then

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r} \frac{d}{dr} \left(r \left(\frac{dv}{dr} \right)^n \right) \quad (15)$$

Analysis

Let $-\left(\frac{dp}{dz}\right) = P$ pressure in the arteries farthest from the heart can be assumed to be constant, equation (15) looks like this:

$$\frac{d}{dr} \left(r \left(\frac{dv}{dr} \right)^n \right) = -\frac{Pr}{\eta_m} (16)$$

On integrating equation (16), we get

$$r \left(\frac{dv}{dr} \right)^n = -\frac{Pr^2}{2\eta_m} + A (17)$$

Apply boundary condition (i) then A = 0

$$r \left(\frac{dv}{dr} \right)^n = -\frac{Pr^2}{2\eta_m} \Rightarrow -\frac{dv}{dr} = \left(\frac{Pr}{2\eta_m} \right)^{1/n} (18)$$

Integrating equation (18) we obtain

$$v = -\left(\frac{P}{2\eta_m} \right)^{1/n} \frac{r^{\frac{1}{n+1}}}{(n+1)/n} + B (19)$$

Apply boundary condition (ii) we obtain

$$B = \left(\frac{P}{2\eta_m} \right)^{1/n} \frac{n R^{1/n+1}}{n+1} \quad (20)$$

Using equation [20] in the equation (19) then

$$v = \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} \frac{n}{n+1} \left[R^{\frac{1}{n+1}} - r^{\frac{1}{n+1}} \right] \quad (21)$$

This is the blood velocity in the arteries.

Then total flow- flux

$$Q = \int_0^R v . 2\pi r dr = \int_0^R \left(\frac{P}{2\eta_m} \right)^{1/n} . \frac{1}{n+1} \left(R^{1/n+1} - r^{1/n+1} \right) 2\pi r dr$$

$$= \left(\frac{P}{2\eta_m} \right)^{1/n} . \frac{2\pi n}{n+1} \left(\frac{R^{1/n+1} . r^2}{2} - \frac{n . r^{1/n+1}}{3n+1} \right)_0^R$$

$$= \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} . \frac{2\pi n}{n+1} . \frac{(n+1) R^{1/n+3}}{2(3n+1)}$$

$$Q = \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} . \frac{\pi n R^{\frac{1}{n}+3}}{(3n+1)} \quad , \quad where P = -\frac{dp}{dz}$$

$$Q = \left[\frac{P_i - P_f}{2\eta_m (z_i - z_f)} \right]^{\frac{1}{n}} . \frac{\pi n R^{\frac{1}{n}+3}}{(3n+1)} (22)$$

Table1: Pathological data of malaria patient for blood

S. N.	Parameter	15/09/2025	17/09/2025	19/09/2025	21/092024	23/09/2025
1	WBC $[\times 10^3/\mu L]$	8.6	8.2	7.8	7.2	7.6
2	Hb[g/dL]	11.5	10.7	9.8	9.2	9.3
3	Platelets $[\times 10^3/\mu L]$	130	98	90	85	110
4	RBC $[\times 10^6/\mu L]$	3.8	3.6	3.3	3.2	3.7

Table2: Pathological blood pressure and hemoglobin

S. N.	Date	B.P(mmHg)	B.P. in Pascal	Cli. BPD	Hb	Hct
1	15/09/2025	140/100	18664.8/13332	2666.4	11.5	34.5
2	17/09/2025	141/102	18798.12/13598.64	2599.74	10.7	32.1
3	19/09/2025	132/95	17331.6/12665.4	2466.42	9.8	29.4
4	21/09/2025	128/94	17064.96/12132.12	2266.44	9.2	27.6
5	23/09/2025	133/97	17998.2/13332	2399.76	9.3	27.9

X = $\frac{3.52\times10^6}{\mu L}$, Y = $\frac{7.88\times10^3}{\mu L}$, Platelet = Y = $\frac{102.6\times10^3}{\mu L}$

$$\eta_m = X\eta_c + Y\eta_w + (1-X-Y)\eta_p$$

Average Systolic pressure =17971.536 Pa

Average Diastolic Pressure = 13012.032 Pa

Pi = Pressure in Artery = Average Systolic Pressure =17971.536 Pa

Pf = Pressure in Arterioles = $\frac{S+D}{2}$ = 15491.784 Pa

H = Percentage of RBC = 30.3

Percentage of WBC = 0.217

According to Glenn Elert (2010) [4]

η_m = Viscosity of mixture = 0.0035 p.s.

According to Gustafson, Daniel R. (1980)

η_p = Viscosity of plasma =0.0015 pascal sec [5]

η_c = Viscosity of red blood cells =0.0075 pascal sec [6]

Blood Pressure Drop $P_i - P_f = 2479.752$

Length of common hepatic arteries = 0.031 m

Since $\eta_m = X\eta_c + Y\eta_w + (1-X-Y)\eta_p$

Or, $\eta_m = \eta_c\frac{H}{100} + Y\eta_w + \eta_p\left(1-\frac{H}{100}-Y\right) where X = \frac{H}{100}0.0035 = 0.0075\frac{30.3}{100} + 0.00217\eta_w + 0.0015\left(1-\frac{30.3}{100}-0.00217\right)$

$\eta_w = 0.0853709677$ pascal-second = viscosity of WBC

Then η_m will be

$$\eta_m = 0.0075\frac{H}{100} + 0.00217\times0.0853709677 + 0.0015\left(1-\frac{H}{100}\right)$$

$$\eta_m = 6\times10^{-5}H + 0.001685255$$

Now from equation (22), Flow flux is given as

$$Q = \left[\frac{P_i - P_f}{2\eta_m(Z_i - Z_f)}\right]^{1/n}\cdot\frac{\pi n R^{\frac{1}{n}+3}}{(3n+1)} where Q$$

$$= 1000\frac{ml}{min} \Rightarrow 0.0167\frac{lit}{sec}$$

$$= 1.67\times10^{-5}m^3/sec$$

$$1.67\times10^{-5} = \left[\frac{2479.752}{2\times0.0035\times0.031}\right]^{\frac{1}{n}}\cdot\frac{3.14\times n\times(0.0025)^{\frac{1}{n}+3}}{(3n+1)}$$

$$340.382166 = (28568.5715)^{\frac{1}{n}}\left(\frac{n}{3n+1}\right)$$

$$n\log\frac{n}{3n+1} - 2.5319668n + 4.45588853 = 0 \quad (23)$$

$$f(x) = x log\frac{x}{3x+1} - 2.5319668x + 4.45588853$$

and $d\,f(x) = log\frac{x}{3x+1} + \frac{1}{3x+1} - 2.5319668$

Solve above equation by Newton- Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

We obtain n= 1.4376

$$P_i - P_f = \left[\frac{(3n+1)Q}{\pi n R^3}\right]^n\left[\frac{2\eta_m\Delta Z}{R}\right]$$

$$P_i - P_f = \left[\frac{(3\times1.4376+1)1.67\times10^{-5}}{3.14\times1.4376\times(0.0025)^3}\right]^{1.4376}\times\frac{2\times0.031\eta_m}{0.0025}$$

$P_i - P_f = 708780.376\,\eta_m$

$P_i - P_f = 708780.376\,(6\times10^{-5}H + 0.001685255)$

$P_i - P_f = 708780.376\,(6\times10^{-5}H + 0.001685255)$

$P_i - P_f = 42.5268226H + 1194.47567 \quad (24)$

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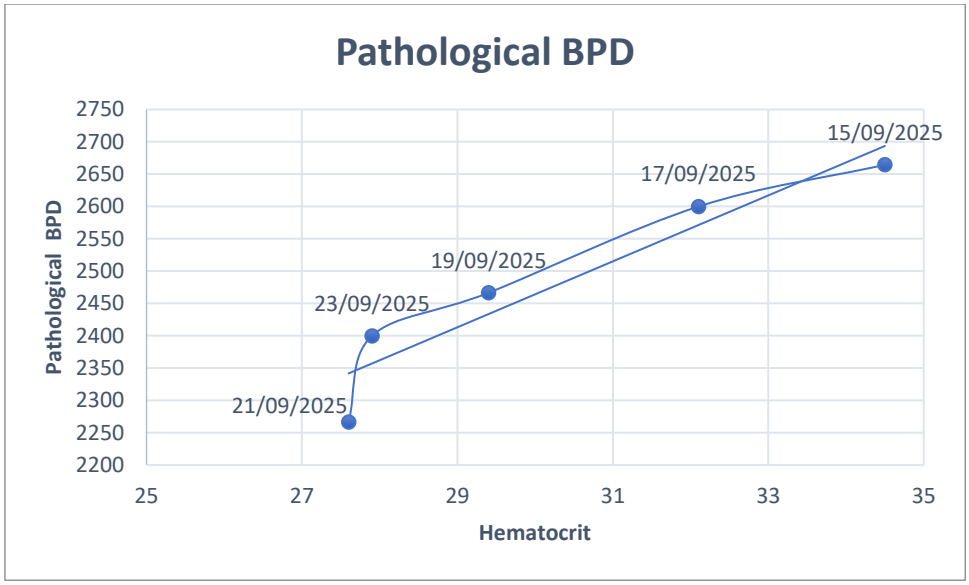
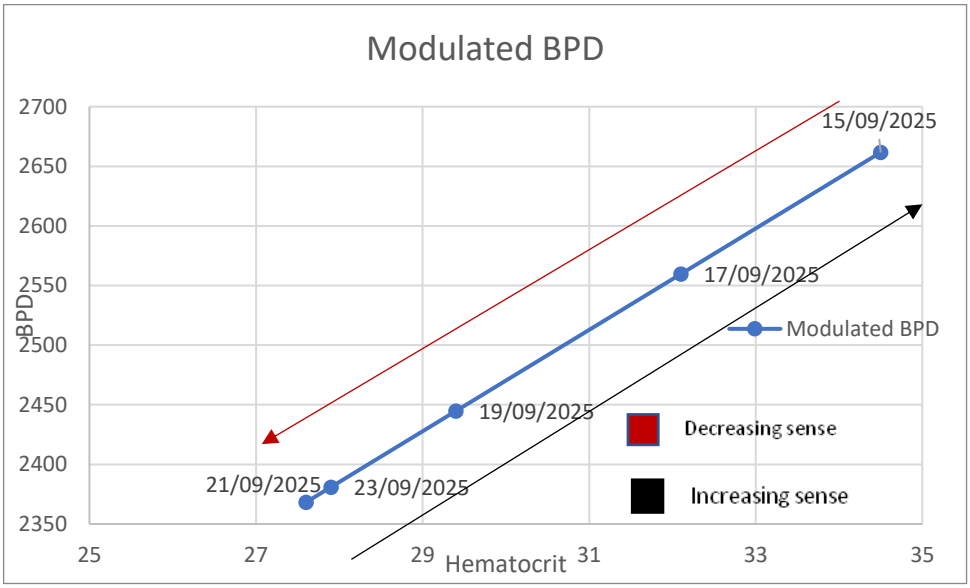
Table-3 Modulated Blood Pressure Drop Vs Hematocrit

Date	15/09/2025	17/09/2025	19/09/2025	21/09/2025	23/09/2025
Hematocrit (H)	34.5	32.1	29.4	27.6	27.9
Blood pressure drop (ΔP)	2661.65	2559.59	2444.76	2368.22	2380.97



Graph 1 Pathological Blood Pressure Drop vs Hematocrit



Graph 2 Modulated Blood Pressure Drop vs Haematocrit

OBSERVATION OF GRAPH

According to Graph **1** it can be shown that the minimum pathological blood pressure (BPD) reaches a value of 2266.44 on September 15, 2025, with a corresponding haematocrit value of 27.6 and the maximum pathological BPD value was recorded as 2666.5 on September 15, 2025, with a haematocrit value of 34.5. There is a decreasing trend in blood pressure drop when the haematocrit value decreases from 34.5 to 27.6 with an intermediate value of 32.1 and 29.4. The haematocrit value decreased from 27.6 to 27.9. This decrease in haematocrit was associated with decrease in blood pressure drop. Additionally, the haematocrit value further decreased from 27.9 to 27.6, resulting in a continued decrease in blood pressure drop.

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Five distinct dates were noted in graph 2 between computed blood pressure drop and haematocrit. The lowest blood pressure drop was 2368.22 on September 21, 2025, while the highest value was 2661.65 on September 15, 2025. From September 15, 2025, to September 21, 2025, via September 17 and 19, the blood pressure drop directly drops at the haematocrit value from 34.5 to 27.6 via 32.1 and 29.4, and from September 21 to September 23, 2025, the blood pressure drop directly increases at the haematocrit value from 27.6 to 27.9.

CONCLUSION

The study found that the blood pressure drops increases with an increase in haematocrit and reduces with a decrease in haematocrit. The slope of the straight line is an absolute number. This suggests that a straight-line trend that decreases sensation causes the dose of medication to gradually grow; a low steepness of curve indicates a high dose of medication; and a straight-line trend that raises sense indicates a typical dose of medication.

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