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GENERALIZED CONTRACTING MAPPING ON M_b-METRIC SPACES WITH APPLICATION

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ABSTRACT

In the present paper, we establish some fixed point theorems in the framework of M_b-metric space. As illustrations few examples are presented. Finally, as application, we discuss the existence of non-linear integral equation solution.
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Keywords: Mapping, theorem, non-linear, equation, generalized contraction mapping etc.

INTRODUCTION

As a generalization of metric space, in 1989, Bakhtin [2] (and Czerwik [3], 1993) derived a number of theorems of fixed points in the form of partial metric spaces. In 1994, Matthews [13] introduced the concept of partial metric space. This extension of metric states that the distance between a point and itself is not zero. The theory of the metric fixed point has been generalised by several researchers in different directions. (see [4-7, 9-12]).

Asadi et al. [1] in 2014 introduced M-metric space, the generalization of partial metric space and produced some fixed-point results on generalized contractions. M_b -metric space was introduced in 2016 (Mlaiki et al., 2016) [14]. This structure is an extension of partial metric space and yields some fixed-point Theorems.

In light of the same spirit, the aim of this paper, is to define generalized contraction map in order to examine the existence of a fixed point for this mapping, in M_b -metric space. In the current literature, our results have extended significantly a number of well documented findings.

2. Preliminaries

Let's start by reviewing the following notation:

Notation 2.1 [1]

1.$m_{\mu,\sigma} = \min\{m(\mu,\mu), m(\sigma,\sigma)\}$

2.$M_{\mu,\sigma} = \max\{m(\mu,\mu), m(\sigma,\sigma)\}$

Definition 2.2. [1] Let φ be a nonempty set. Suppose $m: \varphi^2 \to R^+$ satisfies

$(m1)$ $m(\mu,\ \mu) = m(\sigma,\ \sigma) = m(\mu,\ \sigma)$ if and only if $\mu = \sigma$,

$(m2)$ $m_{\mu,\sigma} \le m(\mu,\ \sigma)$,

$(m3)$ $m(\mu,\ \sigma) = m(\sigma,\ \mu)$,

$(m4)$ $(m(\mu,\ \sigma) - m_{\mu,\sigma}) \le (m(\mu,\omega) - m_{\mu,\omega}) + (m(\omega,\ \sigma) - m_{\omega,\sigma})$ for all $\mu,\sigma,\omega \in \varphi$. Then (φ, m) is called an M-metric space.

The concept of M_b -metric space was given by Mlaiki et al. [14], but first we review the following notation.

Notation 2.3. [14]

(1) $m_{b_{\mu,\sigma}} = min\{m_b(\mu,\mu), m_b\ (\sigma,\sigma)\}$

(2) $M_{b_{\mu,\sigma}} = max\{m_b(\mu,\mu), m_b(\sigma,\sigma)\}$

Definition 2.4. [14] Let φ be a nonempty set. Suppose $m_b: \varphi^2 \to R^+$ satisfies

(m_b1) $m_b(\mu,\ \mu) = m_b(\sigma,\ \sigma) = m_b(\mu,\ \sigma)$ if and only if $\mu = \sigma$,

(m_b2) $m_{b_{\mu,\sigma}} \le m_b\ (\mu,\ \sigma)$,

(m_b3) $m_b\ (\mu,\ \sigma) = m_b(\sigma,\ \mu)$,

(m_b4) $(m_b(\mu,\ \sigma) - m_{b_{\mu,\sigma}}) \le s[(m_b(\mu,\ \omega) - m_{b_{\mu,\omega}}) + (m_b(\omega,\ \sigma) - m_{b_{\omega,\sigma}})] - m_b(\omega,\ \omega)$.

for all $\mu,\sigma,\omega \in \varphi$, where $s \ge 1$, then (φ, m_b) is called an M_b-metric space.

Example 2.5. Let $\varphi = [0,\ \infty)$ and m_b: $\varphi^2 \to \mathrm{R}^+$, for all $\mu,\ \sigma \in \varphi$ we have

$$m_b(\mu,\ \sigma) = |\mu - \ \sigma|^2 + \left(\frac{\mu+\sigma}{4}\right)^2.$$

Note that (φ, m_b) is an M_b-metric space with $s\ = 2$, but it is not M-metric space since the triangle inequality is not satisfied.

Example 2.6. Let $\varphi = [0,\ \infty)$ and m_b: $\varphi^2 \to \mathrm{R}^+$, for all $\mu,\ \sigma \in \varphi$ we have

$$m_b(\mu, \sigma) = |\mu - \sigma|^2 + 3.$$

Note that (φ, m_b) is an M_b-metric space with $s = 2$, but it is not a cone b-metric space over Banach algebra A since for and $\mu > 0$, we have $m_b(\mu, \mu) \neq 0$.

Example 2.7. Let $\varphi = [0, \infty)$ and $m_b: \varphi^2 \to R^+$, for all $\mu, \sigma \in \varphi$ we have

$$m_b(\mu, \sigma) = (max\{\mu, \sigma\})^2.$$

Note that (φ, m_b) is an M_b-metric space with $s = 2$, but it is not M-metric space since the triangle inequality is not satisfied.

Example 2.8.[14] Let $\varphi = [0, \infty)$ and $l > 1$ be constant and $m_b: \varphi^2 \to [0, \infty)$ defined for all $\mu, \sigma \in \varphi$ we have

$$m_b(\mu, \sigma) = max\,\{\mu, \sigma\}^l + |\mu - \sigma|^l.$$

Note that (φ, m_b) is an M_b-metric with s = 2^l, but it is not M-metric space since the triangle inequality is not satisfied.

3. Topology for M_b-metric space

Definition 3.1. [14] Let (φ, m_b) be an M_b-metric space with $s \geq 1$. Then, for all $x \in \varphi$ and $\varepsilon > 0$, the open ball with centre μ and radius ε is defined by

$$B_{m_b}(\mu, \varepsilon) = \{\sigma \in \varphi: m_b(\mu, \sigma) < m_{b_{\mu,\sigma}} + \varepsilon\}.$$

Definition 3.2. Let (φ, m_b) be an M_b-metric space with $s \geq 1$. Each M_b-metric generates a topology τ_{m_b} on φ whose base is the family of open m_b-balls $\{B_{m_b}(\mu, \varepsilon): \mu \in \varphi, \varepsilon > 0\}$, where $B_{m_b}(\mu, \varepsilon) = \{\sigma \in \varphi: m_b(\mu, \sigma) - m_{b_{\mu,\sigma}} < \varepsilon\}$.

Proposition 3.3. An M_b-metric space is a T_0-space.

Proof: Let (φ, τ_{m_b}) be an M_b-metric space and $\mu, \sigma \in \varphi$ such that $\mu \neq \sigma$. Then from $(m_b 2)$, we have

$$m_{b_{\mu,\sigma}} \leq m_b(\mu, \sigma) \Rightarrow$$

$$\min\{m_b(\mu, \mu), m_b(\sigma, \sigma)\} \leq m_b(\mu, \sigma),$$

That is,

$$m_b(\mu, \mu) \leq m_b(\mu, \sigma) \text{ or } m_b(\sigma, \sigma) \leq m_b(\mu, \sigma).$$

Firstly, assume that $m_b(\mu, \mu) = m_b(\sigma, \sigma)$. Then we have

$$m_{b_{\mu,\sigma}} = m_b(\mu, \mu) = m_b(\sigma, \sigma) < m_b(\mu, \sigma),$$

Yielding $m_b(\mu, \sigma) - m_{b_{\mu,\sigma}} = m_b(\mu, \sigma) - m_b(\mu, \mu) > 0$.

If we choose $\varepsilon > 0$ such that $m_b(\mu, \sigma) - m_b(\mu, \mu) = \varepsilon$ then $m_b(\mu, \sigma) < m_{b_{\mu,\sigma}} + \varepsilon$, so that $\sigma \notin B_{m_b}(\mu, \varepsilon)$. Next, assume that $m_b(\mu, \mu) < m_b(\sigma, \sigma)$. Then

$$m_{b_{\mu,\sigma}} = m_b(\mu, \mu) < m_b(\mu, \sigma),$$

$$\Rightarrow m_b(\mu, \sigma) - m_{b_{\mu,\sigma}} = m_b(\mu, \sigma) - m_b(\mu, \mu) > 0.$$

Again, if we choose $\varepsilon > 0$ such that $m_b(\mu, \sigma) - m_b(\mu, \mu) = \varepsilon$, then $m_b(\mu, \sigma) < m_{b_{\mu,\sigma}} + \varepsilon$, so that $\sigma \notin B_{m_b}(\mu, \varepsilon)$.

Similarly, for $m_b(\mu, \mu) > m_b(\sigma, \sigma)$, one can easily show that $\mu \in B_{m_b}(\mu, \varepsilon)$ and $\sigma \notin B_{m_b}(\mu, \varepsilon)$. Therefore, for any two distinct points $\mu, \sigma \in \varphi$, there is a ball containing one and not containing the other point. Hence (φ, m_b) is a T_0-space.

We now discuss the definitions of convergence in M_b-metric space.

Definition 3.4.[14-15] Let (φ, m_b) be a M_b-metric space. Then:

1) A sequence $\{\mu_n\}$ in φ converges to a point μ if and only if

$$\lim_{n,m\to\infty} m_b(\mu_n, \mu_m) - m_{b_{\mu_n,\mu_m}}$$

2) A sequence $\{\mu_n\}$ in φ is said to be M_b-Cauchy sequence if and only if

$$\lim_{n,m\to\infty} (m_b(\mu_n, \mu_m) - m_{b_{\mu_n,\mu_m}}) \text{ and}$$

$$\lim_{n,m\to\infty} (M_{b_{\mu_n,\mu_m}} - m_{b_{\mu_n,\mu_m}}) \text{ exists and finite.}$$

3) An M_b-metric space is said to be complete if every M_b-Cauchy sequence $\{\mu_n\}$ converges to a point μ such that

$$\lim_{n,m\to\infty} m_b(\mu_n, \mu_m) - m_{b_{\mu_n,\mu_m}} = 0 \text{ and}$$

$$\lim_{n,m\to\infty} M_{b_{\mu_n,\mu_m}} - m_{b_{\mu_n,\mu_m}} = 0.$$

MAIN RESULTS

We now state our main results.

Theorem 4.1: Let (φ, m_b) be a complete M_b-metric space with $s \geq 1$ and $\xi: \varphi \to \varphi$ satisfying the condition:

$$(4.1) \qquad m_b(\xi\mu, \xi\sigma) \leq \alpha m_b(\mu, \sigma) + \beta m_b(\mu, \xi\mu) + \gamma m_b(\sigma, \xi\sigma)$$

$\forall$ μ, $\sigma \in \varphi$, where $\alpha, \beta, \gamma, \rho \geq 0$, with $\alpha + \beta + \gamma < \frac{1}{s}$, then ξ has a unique fixed point u such that $m_b(u, u) = 0$.

Proof: Let $\mu_0 \in \varphi$ be arbitrary. Consider the sequence $\{\mu_n\}$ defined by $\mu_n = \xi^n \mu_0$ and $m_{b_n} = m_b(\mu_n, \mu_{n+1})$. Note that if there exists a natural number n such that $m_{b_n} = 0$, then μ_n is a fixed point of ξ. So, assume that $m_{b_n} > 0$, for $n \geq 0$. By (4.1), we have

$$m_{b_n} = m_b(\mu_n, \mu_{n+1}) = m_b(\xi\mu_{n-1}, T\xi) \leq \alpha m_b(\mu_{n-1}, \mu_n) + \beta m_b(\mu_{n-1}, \xi\mu_{n-1}) + \gamma m_b(\mu_n, \xi\mu_n)$$
$$= \alpha m_b(\mu_{n-1}, \mu_n) + \beta m_b(\mu_{n-1}, \mu_n) + \gamma m_b(\mu_n, \mu_{n+1})$$
$$= \alpha m_{b_{n-1}} + \beta m_{b_{n-1}} + \gamma m_{b_n}$$
$$= (\alpha + \beta)\, m_{b_{n-1}} + \gamma m_{b_n}$$

for any $n \geq 0$, $m_{b_n} \leq (\alpha + \beta)\, m_{b_{n-1}} + \gamma m_{b_n}$, which implies $m_{b_n} \leq \rho m_{b_{n-1}}$, where $\rho = \frac{\alpha+\beta}{1-\gamma} < 1$ as $\alpha + \beta + \gamma < \frac{1}{s}$. By repeating this process, we get $m_{b_n} \leq \rho^n m_{b_{n-1}}$. Thus, $\lim_{n\to\infty} m_{b_n} = 0$. By (4.1), for all $n, m > 0$, we have

$$m_b(\mu_n, \mu_m) = m_b(\xi^n\mu_0, \xi^m\mu_0)$$
$$= m_b(\xi\mu_{n-1}, \xi\mu_{m-1})$$
$$\leq \alpha m_b(\mu_{n-1}, \mu_{m-1}) + \beta m_b(\mu_{n-1}, \xi\mu_{n-1}) + \gamma m_b(\mu_{m-1}, \xi\mu_{m-1})$$
$$= \alpha m_b(\mu_{n-1}, \mu_{m-1}) + \beta m_b(\mu_{n-1}, \mu_n) + \gamma m_b(\mu_{m-1}, \mu_m)$$

Thus, from the above inequality, we deduce that

$$m_b(\mu_n, \mu_m) \leq \alpha m_b(\mu_{n-1}, \mu_{m-1}) \text{ for all } n \geq 0.$$

By repeating this process, we get

$$m_b(\mu_n, \mu_m) \leq \alpha^n m_b(\mu_0, \mu_{m-n}) \text{ for all } n \geq 0.$$

Hence,

$$m_b(\mu_n, \mu_m) - m_{b_{\mu_n,\mu_m}} \leq \alpha^n[s m_b(\mu_0, \mu_1) + s m_b(\mu_1, \mu_{m-n})]$$
$$\leq \alpha^n[s m_b(\mu_0, \mu_1) + s^2 m_b(\mu_1, \mu_2) + s^2 m_b(\mu_2, \mu_{m-n})]$$
$$\leq \alpha^n[s m_b(\mu_0, \mu_1) + s^2 m_b(\mu_1, \mu_2) + \cdots + s^{m-n} m_b(\mu_{2m-n-1}, \mu_{m-n})]$$
$$\leq \alpha^n s m_b(\mu_0, \mu_1) + \alpha^n s^2 m_b(\mu_0, \mu_1) + \cdots + \alpha^n s^{m-n} m_b(\mu_0, \mu_1)$$
$$\leq s\alpha^n[1 + s\alpha + (s\alpha)^2 + \cdots] m_b(\mu_0, \mu_1)$$
$$= \frac{s\alpha^n}{1-s\alpha} m_b(\mu_0, \mu_1)$$

As $\alpha < \frac{1}{s}$ and $s > 0$, from the above inequality follows that

$$\lim_{n,m\to\infty} m_b(\mu_n, \mu_m) - m_{b_{\mu_n,\mu_m}} = 0.$$

Similarly, one can show that $\lim_{n,m\to\infty} M_{b_{\mu_n,\mu_m}} - m_{b_{\mu_n,\mu_m}} = 0$.

Thus, $\{\mu_n\}$ is an M_b-Cauchy sequence in φ. Since φ is complete there exist $u \in \varphi$ such that $\lim_{n\to\infty} m_b(\mu_n, u) - m_{b_{\mu_n,u}} = 0$.

Next, we prove that u is a fixed point of ξ. For any $n > 0$, we have $\lim_{n\to\infty} m_b(\mu_n, u) - m_{b_{\mu_n,u}} = 0$

$$= \lim_{n\to\infty} m_b(\mu_{n+1}, u) - m_{b_{\mu_{n+1},u}}$$
$$= \lim_{n\to\infty} m_b(\xi\mu_n, u) - m_{b_{\xi\mu_n,u}}$$
$$= m_b(\xi u, u) - m_{b_{\xi u,u}}$$

which implies that $m_b(\xi u, u) - m_{b_{\xi u,u}} = 0$, hence $m_b(\xi u, u) = m_{b_{\xi u,u}}$, therefore $\xi u = u$. Thus, u is a fixed point of ξ. Now, we show that if u is a fixed point, then $m_b(u, u) = 0$, assume that u is a fixed point of ξ,

hence
$$m_b(u, u) = m_b(\xi u, \xi u) \leq \alpha m_b(u, u) + \beta m_b(u, \xi u) + \gamma m_b(u, \xi u)$$
$$= (\beta + \gamma)\, m_b(u, \xi u) + \alpha m_b(u, u)$$
$$= (\alpha + \beta + \gamma)\, m_b(u, u)$$

As $\alpha + \beta + \gamma < \frac{1}{s}$,

$$\Rightarrow m_b(u, u) = 0.$$

To prove uniqueness, assume that ξ has two fixed points say $u, v \in \varphi$.

Hence
$$m_b(u, v) = m_b(\xi u, \xi v) \leq \alpha m_b(u, v) + \beta m_b(u, \xi u) + \gamma m_b(v, \xi v)$$
$$= \alpha m_b(u, v) + \beta m_b(u, u) + \gamma m_b(v, v)$$
$$\leq \alpha m_b(u, v)$$
$$< m_b(u, v)$$

which implies that $m_b(u, v) = 0$ and thus $u = v$.

Corollary 4.2 Let (φ, m_b) be a complete M_b-metric space with $s \geq 1$ an $\xi: \varphi \to \varphi$ satisfying the condition:

(4.2) $\quad m_b(\xi\mu, \xi\sigma) \leq k m_b(\mu, \sigma)$

$\forall\ \mu, \sigma \in \varphi$, where $k \geq 0$, with $k < \frac{1}{s}$, then ξ has a unique fixed point u such that $m_b(u, u) = 0$.

Example 4.3. Let $\varphi = [0, \infty)$ and $m_b: \varphi \times \varphi \to R$ be defined by $m_b(\mu, \sigma) = |\mu - \sigma|^2 + \left(\frac{\mu+\sigma}{2}\right)^2$. Then (φ, m_b) is a complete M_b-metric space with $s = 2$. Define $\xi: \varphi \to \varphi$ by $\xi\mu = \frac{\mu}{3}$, $\forall\ \mu \in \varphi$.

$$m_b(\xi\mu, \xi\sigma) = |\xi\mu - \xi\sigma|^2 + \left(\frac{\xi\mu + \xi\sigma}{2}\right)^2$$

$$= \left|\frac{\mu}{3} - \frac{\sigma}{3}\right|^2 + \left(\frac{\frac{\mu}{3} + \frac{\sigma}{3}}{2}\right)^2$$

$$= \frac{1}{3^2}|\mu - \sigma|^2 + \frac{1}{3^2}\left(\frac{\mu + \sigma}{2}\right)^2$$

$$= \frac{1}{3^2}\left[|\mu - \sigma|^2 + \left(\frac{\mu + \sigma}{2}\right)^2\right]$$

$$= \frac{1}{9}m_b(\mu, \sigma).$$

Thus, all the conditions of Corollary 4.2 are satisfied with $k = \frac{1}{9}$. Hence ξ has a fixed point $\mu = 0$ and $m_b(0,0) = 0$.

5. Application

In this section, we endeavour to apply Theorem 4.1 to investigate the existence and uniqueness of solution of the Fredholm integral equation.

Consider the following integral equation:

$$(5.1) \qquad \mu(t) = \int_0^{\xi} G\big((t, s, \mu(t))\big)ds,$$

for $t, s \in [0, \xi]$, where $\xi > 0$ and $G: [0, \xi] \times [0, \xi] \times R \to R$. In this section, we present the existence theorem for (5.1). Let $\varphi = C[0, \xi]$ be the set of continuous real functions defined on $[0, \xi]$. We endow φ with the M_b-metric

$$m_b(\mu(t), \sigma(t)) = \sup_{t\in[0,T]} \left(\frac{\mu(t)+\sigma(t)}{4}\right)^2, \text{ for all } \mu, \sigma \in \varphi.$$

Then (φ, m_b) is a complete M_b-metric space with the constant $s = 2$.

Let $f(\mu(t)) = \int_0^{\xi} G(t, s, \mu(t))ds$ for all $\mu \in \varphi$ and for all $t, s \in [0, \xi]$. Then the existence of a solution to (5.1) is equivalent to the existence of a fixed point of f. Now, we prove the following result.

Theorem 5.1: Assume that for all $\mu, \sigma \in C[0, \xi]$

$$(5.2) \qquad |G(t, s, \mu(t)) + G(t, s, \sigma(t))| \leq \lambda^{\frac{1}{2}}|\mu(t) + \sigma(t)|$$

for all $t, s \in [0, \xi]$ where $0 < \lambda < \frac{1}{s}$. Then the integral equation (5.1) admits a unique solution in $\mu \in \varphi$.

Proof. From (5.2), for all $t \in [0, \xi]$, we have

$$m_b(\xi\mu(t), \xi\sigma(t)) = \left(\frac{\xi\mu(t) + \xi\sigma(t)}{4}\right)^2$$

$$= \left|\int_0^{\xi} \left(\frac{K(t,s,\mu(t))+K(t,s,\sigma(t))}{4}\right) ds\right|^2$$

$$\leq \int_0^{\xi} \left|\left(\frac{K(t,s,\mu(t))+K(t,s,\sigma(t))}{4}\right)\right|^2 ds$$

$$\leq \int_0^{\xi} \left\{\lambda^{\frac{1}{2}} \left|\left(\frac{\mu(t)+\sigma(t)}{4}\right)\right|\right\}^2 ds$$

$$\leq \lambda \int_0^{\xi} \left\{\left(\frac{|\mu(t)|+|\sigma(t)|}{4}\right)^2\right\} ds$$

$$\leq \lambda m_b(\mu(t), \sigma(t)).$$

Thus, condition (5.1) is satisfied. Therefore, all conditions of Theorem 4.1 are satisfied. Hence ξ has a unique fixed point, which means that the Fredholm integral equation (5.3) has a unique solution. This completes the proof.

Open Problems: Prove analogue of Reich contraction, Ciric contraction and Hardy-Rogers contraction in M_b-metric space.

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